# An Experimental Analysis of Blockchain Mining Using Game Theory

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***Abstract*—Mining fairness in blockchain is a crucial factor that affects the sustainability and motivation of proof-of-work calculations. If fairness declines, rational miners may be discouraged from participating, leading to reduced computational resources in the network. At the same time, blockchain transaction processing capacity is in a dilemma with mining fairness, making a deep understanding of these relationships essential for maintaining a highly available and well-functioning network.**

**This study models mining as a game and analyzes how rational miners' strategies impact network-wide mining fairness using game theory. Miners adjust their hash rate to maximize rewards, sometimes adopting extreme strategies like committing all resources or exiting entirely. A large-scale simulation extends this model to more complex network structures, where miners stochastically refine their strategies based on their own and others' gains. The findings show that mining fairness is influenced by network topology, transaction throughput, and decentralization indicators such as hash rate bias and network sparsity. Notably, a denser network leads to fairer mining outcomes.**

**By integrating prior quantitative fairness evaluations with game theory and network analysis, this study enables a more detailed and comprehensive understanding of mining fairness, offering valuable insights for blockchain design and optimization.**

## I. Introduction

Blockchain is a technology that maintains a ledger among network participants in a decentralized manner, verifies and approves transactions throughout the network to ensure their legitimacy. Mining plays an important role for maintaining security and reliability of a blockchain.

The core of mining is a mechanism called Proof of Work, which verifies and agrees on transactions through a large number of calculations. Participants, called miners, compete to solve a mathematical problem by using their advanced computing power to perform an enormous amount of hash calculations. The first miner to solve the problem is entitled to generate a new block and approve the transactions contained in that block. Once a new block is generated, it is shared throughout the network and verified by other nodes.

Miners who solve the problem are rewarded with cryptocurrency. This reward consists of newly generated coins and transaction fees within the block. In this way, miners are incentivized to contribute to the network while at the same time being supplied with new currency.

Blockchain, however, has been proposed with the concept of mining fairness, a concept that undermines the fundamental mechanism of this technology. Ideally, blockchains aim to have all blocks in a single chain, in which all miners are rewarded according to the computational resources they have invested. In reality, however, factors such as network delays and attacks by malicious participants can cause forking, where multiple blocks are created at the same time and the chain splits [1]–[4]. When a fork occurs, each participant mines the block that it believes will become the main chain, but not all blocks are incorporated into the main chain. However, only the blocks in the main chain are rewarded when mining succeeds, resulting in the generation of computational resources that are not rewarded even though they have been invested. The equality of the computational resources and the obtained block rewards is mining fairness, and the degradation of mining fairness refers to the discrepancy between the computational resources invested and the generated rewards.

This situation leads to the departure of miners with low expected profit margins and, conversely, to the concentration of rewards on miners with high profit margins, thus undermining the decentralized nature of the chain. Therefore, a decrease in mining fairness is a situation that should be avoided for the sound operation of the system. On the other hand, it is known that increasing the transaction processing capacity of a chain increases the forking rate of the chain [5], [6], which leads to a decrease in mining fairness. Therefore, these characteristics are in a dilemma and cannot be reconciled.

As mentioned above, the relationship between mining fairness and transaction processing capacity is an important indicator for chain characterization and has attracted much attention [7]–[10]. Therefore, in this study, we model these relationships as a game of miners who change the amount of computational resources they invest as a strategy, and aim to understand how these characteristics interact in a real chain.

## II. Related Work

Mining fairness refers to the equality between the computational resources invested and the rewards obtained. Our previous study [11] proposed a method for quantifying it. In this study, mining fairness is expressed in terms of several quantitative indices, and the calculation method for mining fairness in this study follows the indices of the previous study.

First, this study adopts the local mining fairness index $LF_1$ metric for quantitative assessment of mining fairness. The $LF_1(i)$ represents the difference between the block reward rate of minor i and the hash rate percentage of the network.

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Here, let round $r$ be the time interval between the first generation of a block of height $r$ and the first generation of a block of height $r+1$, and let $f$ the probability that $j$ forks when round $r$ begins with the generation of a block of minor $i$ and minor $j$ generates a block. Furthermore, let $T$ be the average block generation interval and $d$ be the block propagation time between minors.

According to this study, $LF_1$ in the network with multiple miners can be obtained as where $V$ is the set of all the miners and $\alpha_i$ is the hash rate ratio of minor $i$ in the whole network. $\pi$ in the following equation denotes the stationary distribution of minors starting a round when enough time has elapsed. The $LF_1$ for node $i$ is obtained as follows.

$$\begin{aligned} LF_1(i) = \pi(i)(1 - \sum_{j \in V} \alpha_j F_{ij} + \sum_{j \in V} \alpha_j F_{ij} W_{ij}) \\ + \sum_{j \in V} \pi(j) \alpha_i F_{ji} (1 - W_{ji}) - \alpha_i \end{aligned} \tag{1}$$

Here, the fork ratio $F_{ij}$ is obtained by using the block propagation time $T_{ij}$ between nodes $i$ and $j$ and the average block generation interval $T$ as follows.

$$F_{ij} = 1 - e^{-\frac{T_{ij}}{T}} \tag{2}$$

Since $W_ij$ is determined by the chain conflict resolution rule, we use the first-seen rule adopted in Bitcoin, i.e., each miner mines the first block it receives. Let $P_{i,j,k}$ denote the probability that minor $k$ mines minor $i$'s block when minor $i$ starts a round and minor $j$ has a chain conflict, and $T_{ij}$ denote the block propagation time from minor $i$ to $j$, where $P_{i,j,k}$ and $W_ij$ are obtained as follows.

$$P_{i,j,k} = \begin{cases} 1 & \text{if } T_{ik} < T_{jk}, \\ 0 & \text{else if } T_{ik} < T_{ij} + T_{jk}, \\ \frac{e^{-\frac{T_{ik}-T_{jk}}{T}} - e^{-\frac{T_{ij}}{T}}}{1 - e^{-\frac{T_{ij}}{T}}} & \text{else.} \end{cases} \tag{3}$$

$$W_{ij} \approx \sum_{k \in V} \alpha_k P_{i,j,k} \tag{4}$$

Furthermore, we introduce $LF_2$ as an indicator of the profit margin of each minor. The $LF_2$ for node $i$ is obtained as follows. It is the gain of node $i$ in game theoretic terms in this study.

$$LF_2(i) = \frac{LF_1(i)}{\alpha_i} \tag{5}$$

## III. Our Approach

In order to better understand the rational behavior of miners in terms of mining fairness, We conducted a simulation of a complex network with a large number of miners.

First, to start the simulation, we assume a network with multiple random points in a unit square of length one, such that the Euclidean distance between these nodes is the block propagation time between each node.

Then, we create a network connecting these nodes as a relation of the target that these nodes imitate when updating their strategies. The network connecting these nodes is created. In this implementation, we used the Barabási-Albert model [12], which is a typical model for generating scale-free networks.

Next, we assign a hash rate to each node as a strategy. As the initial value, we take a random value following a normal distribution.

Since $\pi$ represents the stationary distribution of minors starting a round after enough time has passed, it can be obtained by repeating rounds until this distribution converges. From the above, the value of $LF_1(i)$ is obtained, and $LF_2(i)$ is also obtained by dividing it by $\alpha_i$, the hash rate ratio of each node.

In this study, we applied a model called Pairwise Comparison [13] to update the strategy of each node in the simulator. However, in this game, it is obvious that if we completely imitate the strategy taken by the opponent, i.e., if we take the exact same hash rate, all the nodes will immediately converge to the same hash rate, and it is not suitable for the design of the gain function in this game, where a larger gain is obtained by taking a larger hash rate than that of the opponent. Therefore, in this simulator, we will update the strategy by increasing the hash rate by a certain amount if the hash rate of the imitator is larger than our own, and by decreasing the hash rate by the same amount if the hash rate is smaller.

More specifically, the strategy imitation probability $p$ is assumed to follow a Fermi distribution function following the previous study [14][15][16], whose value is expressed as follows.

$$p = \frac{1}{1 + e^{\beta(u_A - u_B)}} \tag{6}$$

where the inverse temperature parameter $\beta$ in the equation represents the intensity of strategy update for each minor, and takes non-negative values. The larger $\beta$ is, the more $p$ varies with the difference of gains, and the smaller $\beta$ is, the more random the strategy updates are. When $\beta = 0$, each minor updates its strategy with probability 1/2 regardless of its gain. In this case, this value is set to 200, and the update range of the strategy is set to 2% of the mean value of the normal distribution given as the initial value.

To evaluate the mining fairness of the entire network, we now introduce a global mining fairness index $GF$. It represents the difference between the maximum and minimum values of $LF_2$ in the network.

The hash rate of each node is initialized according to a normal distribution with mean $400{,}000$ and standard deviation $100{,}000$, and is assumed to be updated in the range of $8{,}000$ when the strategy is updated.

Also, the number of nodes in the network is assumed to be $20$ and the average degree is assumed to be $8$.

## IV. Results

### A. Simulation with fixed network topology

First, in order to evaluate the effect of the average block generation interval on the network consisting of multiple

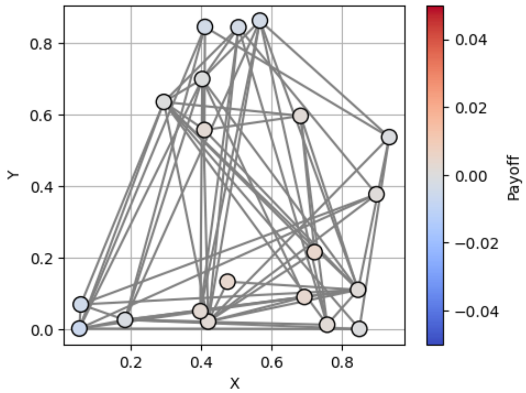


Fig. 1. Average gain $LF_2$ of nodes ($\frac{\Delta}{T} = 0.01$)

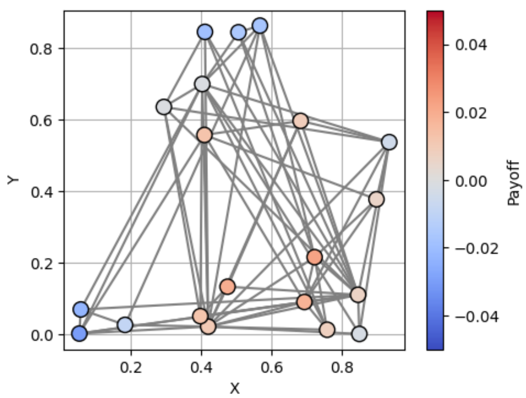


Fig. 2. Average gain $LF_2$ of nodes ($\frac{\Delta}{T} = 0.05$)

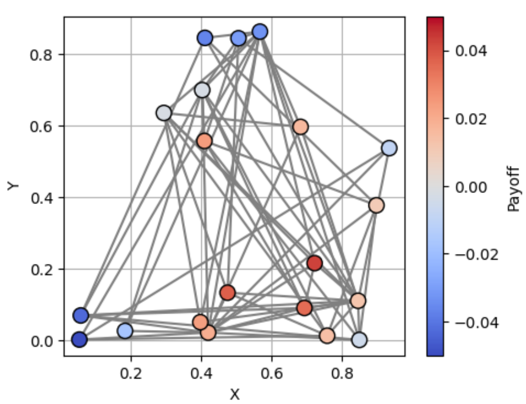


Fig. 3. Average gain $LF_2$ of nodes ($\frac{\Delta}{T} = 0.1$)

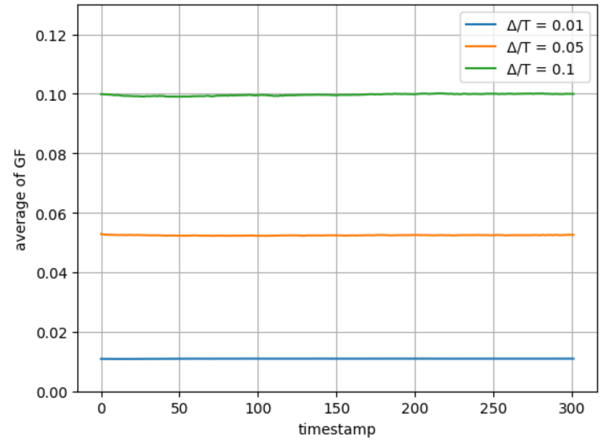


Fig. 4. Mean of $GF$.

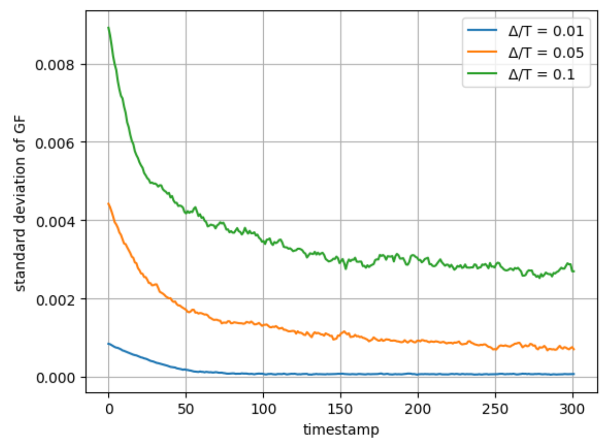


Fig. 5. Standard deviation of $GF$.

nodes, we fix the network topology and the coordinates of each node, and then vary the average block generation interval to evaluate the gain of each node and the global mining fairness of the network.

The average block generation interval is the average distance between each node, i.e., the average block propagation time, as shown in equation 7, and three values: 100 times, 20 times, and 10 times the average block generation interval. These correspond to $\frac{\Delta}{T} = 0.01, 0.05, 0.1$, respectively.

$$\Delta = \frac{\sqrt{2} + 2 + 5\ln(1+\sqrt{2})}{15} \approx 0.521 \qquad (7)$$

For each of these values, we conducted 100 sets of trials, each set consisting of 300 strategy updates.

For each of these values, 100 sets of trials were conducted, each set consisting of 300 strategy updates. The final states of the gains are averaged for each of them, and are shown in Figures 1, 2 and 3.

From these results, we can see that the average block generation interval affects the mining fairness of the entire network. The smaller $\frac{\Delta}{T}$ is, the closer each node's gain is

TABLE I

Pearson correlation coefficient of centrality index and gain at each average block generation interval

| $\Delta/T$ | **Degree** | **Closeness** | **Betweenness** | **Eigenvector** |
|---|---|---|---|---|
| 0.01 | 0.12593 | 0.12885 | 0.14525 | 0.12395 |
| 0.05 | 0.11579 | 0.12221 | 0.13169 | 0.11508 |
| 0.1 | 0.11272 | 0.11323 | 0.12728 | 0.11391 |

TABLE II

Pearson correlation coefficient $p$ values of centrality index and gain at each average block generation interval

| $\Delta/T$ | **Degree** | **Closeness** | **Betweenness** | **Eigenvector** |
|---|---|---|---|---|
| 0.01 | 1.5967E-08 | 7.3401E-09 | 6.7385E-11 | 2.6805E-08 |
| 0.05 | 2.0721E-07 | 4.1917E-08 | 3.3888E-09 | 2.4630E-07 |
| 0.1 | 4.3318E-07 | 3.8375E-07 | 1.1168E-08 | 3.2635E-07 |

to zero, which indicates that the entire network is moving toward a more fair direction. Here, a decrease in $\frac{\Delta}{T}$, i.e., an increase in the average block generation interval $T$, indicates a decrease in transaction processing power, and the results obtained quantitatively show the dilemma relationship between these characteristics.

In this regard, we will evaluate this point using the global mining fairness $GF$. The time-series variation of the mean and standard deviation of $GF$ are shown in Figures 4 and 5.

As expected, the value of GF also decreases as the value of $\Delta/T$ decreases. In addition, a similar trend is observed for the standard deviation, and its effect is also observed for the speed of convergence.

Next, Pearson's correlation coefficient is calculated between the network centrality and the final gain of each node in order to measure the effect of network topology on the gain. The network centrality indices introduced here are order centrality, proximity centrality, vector centrality, and eigenvector centrality [17]–[19].

Tables I and II show the results. From these results, it could not be shown that there is a significant positive correlation between each centrality index and the gains in all cases. However, it is found that the $p$ values are sufficiently small in all cases, and the correlation coefficients are consistently around 0.12.

In conclusion, for this simulator, the effect of the network topology on the gain of each node is not strong, and the effect of the convergence of the mining fairness of the entire network due to the change in the hash rate ratio caused by the strategy update of each node is strongly shown.

### B. Simulation of randomly generated networks

Next, in order to evaluate the impact of network topology on mining fairness based on the results of the previous section, we consider randomly generated network topologies. In the simulations of the randomly generated network, the average block generation intervals are set to be $\frac{\Delta}{T} = 0.01, 0.05, 0.1,$ as in the previous simulation, and for each of these values, 100 sets of trials were conducted for each of these values, with 300 strategy updates as one set.

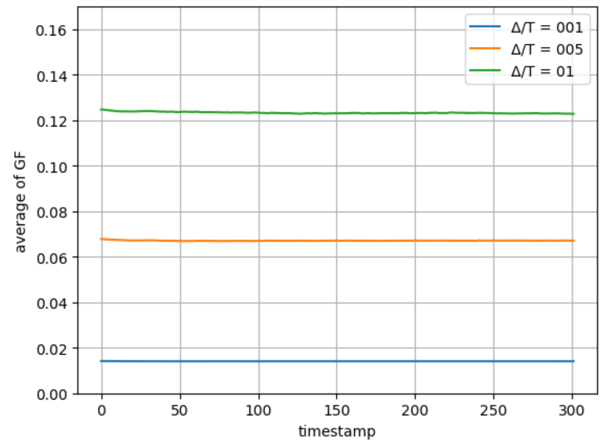


Fig. 6. Mean of $GF$ with a randomly generated network.

TABLE III

Pearson correlation coefficients of centrality indices and gains per average block generation interval with randomly generated networks.

| $\Delta/T$ | **Degree** | **Closeness** | **Betweenness** | **Eigenvector** |
|---|---|---|---|---|
| 0.01 | -0.00539 | -0.00971 | -0.00136 | -0.00707 |
| 0.05 | -0.05016 | -0.04923 | -0.05328 | -0.04246 |
| 0.1 | -0.00065 | 0.00837 | 0.00583 | 0.00214 |

TABLE IV

Pearson correlation coefficient $p$ values of centrality index and gain per average block generation interval with randomly generated networks.

| $\Delta/T$ | **Degree** | **Closeness** | **Betweenness** | **Eigenvector** |
|---|---|---|---|---|
| 0.01 | 0.80950 | 0.66439 | 0.95152 | 0.75200 |
| 0.05 | 0.02488 | 0.02770 | 0.01717 | 0.05765 |
| 0.1 | 0.97686 | 0.70843 | 0.79430 | 0.92385 |

First, in checking the gains of multiple networks, we observed that the gains of nodes located in the center of the network tend to be higher, while the gains of nodes located at the edges tend to be lower. This leads to the assumption that there is a correlation between network centrality and node gains.

The time-series variation of the mean of $GF$ shown in Figure 6 is different from that shown in Figure 4, which leads to the assumption that the converged value of $GF$ depends on its network topology.

In order to verify these assumptions, we first evaluate the correlation between each centrality index and gain as in the previous section. In the tables III and IV, we show the Pearson correlation coefficients and $p$ values for each average block generation interval, each centrality index and gain.

From the above results, either the $p$ values are large and above the significance level and no significant conclusion can be obtained, or the correlation coefficients are very small and no correlation can be obtained even if the $p$ values are below the significance level.

Next, as a property of the network as a whole, we evaluate

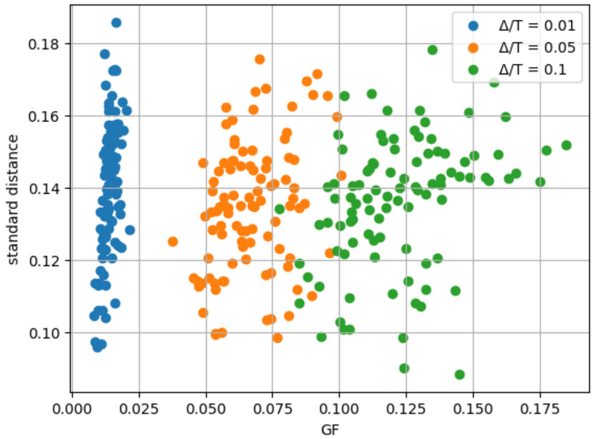


Fig. 7. Converged value of $GF$ and standard distance.

TABLE V
PEARSON CORRELATION COEFFICIENT AND $p$VALUE OF $GF$ WITH STANDARD DISTANCE PER $\Delta/T$.

| $\Delta/T$ | Corr | p |
|---|---|---|
| 0.01 | 0.51210 | $5.1523\times10^{-8}$ |
| 0.05 | 0.29664 | 0.002728 |
| 0.1 | 0.34879 | 0.00037589 |

the relationship between the convergent value of $GF$ and the topology of the network. As a measure of network sparsity, we adopt the standard distance, i.e., the standard deviation of the distance from $(0.5, 0.5)$, the center of gravity of the region. Figure 7 shows the relationship between the convergence value of $GF$ and the standard distance.

In addition, the correlation between the standard distance and $GF$ for each $\Delta/T$ is shown in tableV.

The above results show that there is a significant positive correlation between the standard distance and $GF$ for all average block generation intervals. This confirms that the convergence of $GF$ depends on the network topology, and that the smaller the standard distance, i.e., the more dense and decentralized the network is, the smaller the convergence value of $GF$ is.

## V. CONCLUSION

We conducted simulation to study the rational behavior of miners in a game in which mining fairness is taken into account in the gain, and the rational behavior of miners in a game in which they aim to be rewarded efficiently.

As a result, while confirming that the dilemma between transaction throughput and mining fairness holds even in complex networks, we verify that the converged value of $GF$ depends on the network topology, and that a dense and decentralized network with small centrality and hash rate bias, small standard distance We have verified that a network with small centrality and hash rate bias, small standard distance, and dense decentralized network can construct a healthy network that is fairer and converges more easily. When the bias of centrality and hash rate is large, the variance of $GF$ increases, and the value of $GF$ that converges increases for networks with large delay among sparse nodes.

The expected next step is to conduct theoretical analyses to support the results. Furthermore, as a future prospect, it is possible that more significant results can be obtained by using a distance weighted by hash rate instead of the standard distance as a measure of the geographical closeness of the network. In addition, in the actual blockchain, the difficulty of calculation is always adjusted according to the total amount of hashrate in the network, and reflecting this in the game may provide a different perspective.

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