Correlation Analysis of Reward Rate in a DPoS Blockchain

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***Abstract*—In the blockchain architecture, there are various consensus methods, such as Proof of Work (PoW), Proof of Stake (PoS) and Delegated Proof of Stake (DPoS) for validator selection. A DPoS protocol works similarly to an election, where token holders vote for validator candidates. Elected validators generate blocks and receive block generation rewards. BNB Chain adopting DPoS has implemented a leveling mechanism to prevent the concentration of votes on a small number of validators. However, voting token holders do not always act in economically rational ways. This causes disparity in rewards to votes by token holders. We investigated the BNB Chain and factors related to the reward rates. We calculated Spearman's rank correlation coefficient between reward rates and several factors: the number of delegate-related transactions, the amount of delegated tokens, the rate of change in the amount of delegated tokens, and the average number of voting validators. As a result, we found that the number of delegate-related transactions has the strongest correlation with reward rates, and that there is a non-linear correlation between the reward rate and the average number of voting validators.**

***Index Terms*—Blockchain, Delegated Proof of Stake, BNB Chain, Correlation coefficient**

1. Introduction

Following the advent of Bitcoin in 2008 [1], the core technology, blockchain, has been employed in a variety of fields, including finance [2], healthcare [3], and the Internet of Things [4]. One of the basic functions of the blockchain is the consensus protocol. The consensus protocol defines the rules for verifying transactions and creating a block. The first blockchain Bitcoin employs Proof of Work (PoW) protocol. Subsequently, Proof of Stake (PoS) and Delegated Proof of Stake (DPoS) protocols were developed for other blockchains [5]. DPoS is an election-like protocol where token holders vote for validator candidates, and selected validators generate a block and distribute the rewards earned. In this paper, token holders who have voted are referred to as *delegators*. Compared to the previous consensus protocols such as PoW, DPoS does not waste energy [6] and improves transaction throughput [7]. The DPoS was first introduced in 2014 [8]. Since then, it has been employed by many successful blockchains, including EOSIO [9], Tron [10], and BNB [11].

The BNB Chain employs a mechanism to balance the number of votes for each validator in order to control the concentration of votes on some validators. This mechanism is called a leveling mechanism. The leveling mechanism provides economic incentives to vote for validators with fewer votes. In theory, if delegators act with complete economic rationality, they receive the same rate of reward. In practice, delegators do not act with economic rationality, which results in a disparity in the reward rates they receive. This disparity is caused by the election-like process in DPoS, while no such disparity occurs in PoS. Tanaka et al. [12] analyzed the relationship between the reward rate and the number of deletage-related transactions. However, other factors related to reward rates are not clear.

This paper examines the relationship between the reward rate and four attributes of a delegator: the number of delegate-related transactions, the amount of delegated tokens, the rate of change in the amount of delegated tokens, and the average number of voting validators. The results show that the number of delegate-related transactions has the strongest correlation with the reward rate. In addition, the results show that delegators with an average number of voting validators between two and three showed a higher reward rate than those with other average numbers.

2. BNB Chain

2.1. Overview

The BNB Chain is one of the most popular blockchains in the world and its market cap is reaching 90 billion dollars[1]. BNB plays a significant role in the field of decentralized finance, with the second highest volume of transactions on decentralized exchanges after Ethereum.

The BNB Chain is composed of two blockchains: the BNB Beacon Chain (BC) and the BNB Smart Chain (BSC)[2].

1. https://coinmarketcap.com/ (accessed 2024-04-24)
2. https://docs.bnbchain.org/docs/overview/ (accessed 2024-04-24)

The BC is the blockchain component responsible for the governance of the BNB Chain and manages voting on the BNB Chain. In contrast, the BSC is a blockchain component of that is Ethereum Virtual Machine compatible, consensus layers, and with hubs to multi-chains.

2.2. Delegated Proof of Stake

In the BNB Chain, validators are selected according to the DPoS protocol. First, BNB token holders vote for validator candidates. The elected validators then generate blocks and subsequently receive rewards. Finally, the delegators who have voted for the elected validators are then distributed rewards by the validator.

Token holders can vote for any number of validator candidates with any amount of tokens. The total amount of tokens voted is counted daily at 00:00 UTC, and the top 21 validator candidates are selected as the cabinet, while the top 22–29 validator candidates are selected as the candidate [13][3]. Subsequently, in each epoch, the probability of a cabinet being selected as a validator is higher than that of a candidate. This results in cabinets receiving more rewards and candidates receiving relatively fewer rewards.

Blocks are generated every three seconds. Validators receive a block generation reward from the gas fees of the transactions included in the generated block. Each validator sets its own commission rate, and the validator distributes the block reward minus the commission to the delegators who voted. The reward received by delegator i from validator v on day d is calculated using the following equation:

$$R_{i,d} = \left(R_{v,d} \times (1 - C_{v,d})\right) \times \left(\frac{V_{i,d}}{V_{v,d}}\right), \tag{1}$$

where $R_{v,d}$ is the block generation reward of validator v earned on day d, $C_{v,d}$ is the commission rate of validator v on day d, $V_{i,d}$ is the amount of tokens delegator i voted for validator v on day d, and $V_{v,d}$ is the total amount of tokens voted for validator v on day d.

Assuming that all delegators act economically rationally, the average reward rate should be the same regardless of which validator they vote for. Validators generate blocks with the same probability regardless of the number of votes they receive, which means that all validators earn the same average reward. Since every cabinet is selected as a validator with the same probability, the expected rewards earned by the cabinet are equal. The same can be said for candidates. This leveling mechanism creates an economic incentive to vote for the validator candidates with the low number of votes. This is because, assuming that $R_{v,d}$ is constant and $C_{v,d}$ is equal across validators, the validator with fewer $V_{v,d}$ has a larger $R_{i,d}$. This leveling mechanism equalizes the number of validators' votes and also avoids the concentration of votes on a small number of validators and prevents delegators with a large number of tokens from having an excessive impact on the election.

However, the number of delegators who act in an economically rational manner is unclear. Furthermore, changes in the number of validators or alterations to the election process would result in disparities in the number of votes received. It is important that the delegators exist who maximize their rewards by acting economically rational. Such delegators contribute to maintaining a balance in the number of votes between validators.

3. Related Work

Since the development of DPoS, numerous studies have been conducted regarding attacks specific to DPoS. Hasanova et al. [14] highlighted the risks of a large token holder taking over the election under low voter participation rates and collusion among validators as vulnerabilities specific to DPoS blockchains. In this context, Liu et al. [15] investigated EOS, one of the DPoS blockchains, and revealed that influence is concentrated on some delegators and there are suspicions of collusion among some validators. Furthermore, Wang et al. [16] proposed a method to give more rewards to nodes that actively participate in voting, by classifying voting nodes using a clustering algorithm to mitigate such risks. This method intentionally created a difference in rewards and encouraged nodes to participate in voting. However, in the BNB Chain, not only the voting participation rate but also the imbalance in the number of votes for validators can lead to vulnerabilities.

There is a study by Tanaka et al. [12] that examined the imbalance in the number of votes as other than attacks in DPoS. They examined the relationship between reward rates and voting behavior and showed that delegators who frequently generate delegate-related transactions receive more rewards. However, the relationship between reward rates and attributes other than the number of delegate-related transactions is unclear. In this paper, we examine the relationship between reward rate and four delegator attributes.

4. Data

4.1. Delegated Token Amount

In order to analyze the disparity in reward rates, we collect delegate-related transactions. From these transactions, we calculate the amount of delegated tokens and rewards for each delegator.

We collected delegate-related transactions from BNB Chain Explorer[4]. The three types of delegate-related transactions are as follows:

Delegate Smart Chain Validator
: Vote for delegator candidates using BNB tokens.

Redelegate Smart Chain Validator
: Change the vote from one delegator candidate to another.

3. These numbers are current at the time of data collection.

4. https://explorer.bnbchain.org/ (accessed 2024-04-15)

Undelegate Smart Chain Validator
Cancel the current vote.

Finally, we collected 709,133 delegate-related transactions generated between August 16, 2020 and May 31, 2023.

The amount of delegated tokens is calculated using timestamp, delegator_address, validator_address, delegation_amount in the Delegate Smart Chain Validator and Undelegate Smart Chain Validator transactions. The Redelegate Smart Chain Validator transaction does not affect the total amount of delegated tokens, and thus it is not included in the calculation.

The amount of delegated tokens of delegator i on day d is calculated using the following equation:

$$V_{i,d} = A_{\text{delegate},i,d} - A_{\text{undelegate},i,d}, \quad (2)$$

where $A_{\text{delegate},i,d}$ is the sum of the tokens in the Delegate Smart Chain Validator transactions generated by delegator i before day d, and $A_{\text{undelegate},i,d}$ is the sum of the tokens in the Undelegate Smart Chain Validator transactions generated by delegator i before day d. In addition, the amount of delegated tokens of delegator i in month m, denoted by $V_{i,m}$, is calculated as the sum of $V_{i,d}$.

4.2. Reward Rate

In order to calculate the reward received by each delegator, we collect reward transactions. First, we calculated the total amount of delegated tokens for each delegator as of May 31, 2023. Then, we determined the top 1,000 delegators as the collection target. Finally, we collected the reward transactions for the top 1,000 delegators. This resulted in 1,616,939 reward transactions.

Next, we calculate the monthly reward rate for each delegator. The total reward received by delegator i in month m is calculated as the sum of tokens in reward transactions generated in month m. The monthly reward rate was calculated by excluding delegators who had a monthly delegated token volume of zero.

The monthly reward rate of delegator i in month m is calculated as follows:

$$r_{i,m} = \frac{R_{i,m}}{V_{i,m}} \times \left(\frac{365}{N_m}\right) \times 100, \quad (3)$$

where $R_{i,m}$ is the sum of rewards received by delegator i, $V_{i,m}$ is the total amount of delegated tokens of delegator i in month m, and N_m is the number of days in month m. The value of $r_{i,m}$ is calculated on the assumption that the reward and the amount of delegated tokens of delegator i will continue for one year. Consequently, the formula is multiplied by $\frac{365}{N_m}$. The monthly reward rate was calculated between January 2022 and May 2023.

4.3. Delegator Attributes

Four attributes of delegators were selected for analysis: the number of delegate-related transactions, the amount of delegated tokens, the rate of change in the amount of delegated tokens, and the average number of voting validators.

The number of delegate-related transactions is defined as the total number of delegate-related transactions generated by the delegator in a given month. The existence of the leveling mechanism in the BNB Chain implies that the reward rate may be reduced if the delegator fails to appropriately change a validator with a low reward rate to another when the amount of delegated tokens of the validator is changed. To change the delegating validator, it is necessary to generate a delegate-related transaction. Consequently, it is expected that delegators who generate more delegate-related transactions will have a higher reward rate.

The amount of delegated tokens, denoted by $V_{i,m}$, is defined in Section 4.1. The phenomenon of the rich getting richer has been a topic of discussion in PoS blockchains for some time [17]. Our hypothesis was that a similar phenomenon might occur in DPoS-based BNB Chains, where delegators with larger amounts of delegated tokens may have higher reward rates.

The rate of change in the amount of delegated tokens of delegator i in month m is defined as follows:

$$\text{CR}_{i,m} = \frac{V_{\text{end},i,m} - V_{\text{start},i,m}}{V_{\text{start},i,m}}, \quad (4)$$

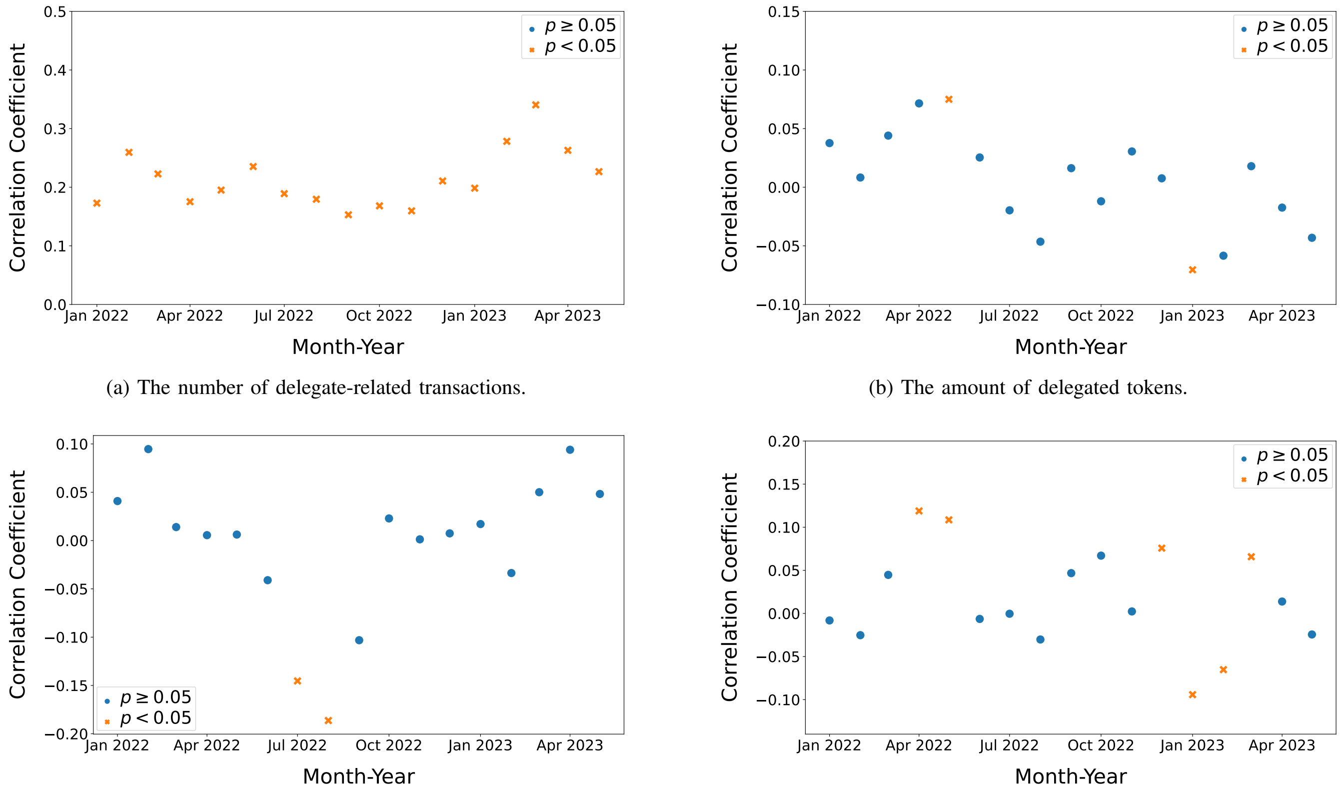
where $V_{\text{start},i,m}$ is the amount of delegated tokens of delegator i at the beginning of month m and $V_{\text{end},i,m}$ is the amount of delegated tokens of delegator i at the end of month m. In the BNB Chain, delegators must cast as many tokens as possible in order to maximize the rewards they receive. We hypothesized that delegators who increase the amount of delegated tokens may be more conscious of profitability, and therefore select the most suitable validator candidates, resulting in a higher reward rate.

The average number of voting validators is the average number of daily voting validators whose votes were cast by delegator i in month m. This value must be at least one. Voting for validator candidates has an investment-like aspect, where the reward rate is determined by the validators they vote for. In general, investment diversification tends to reduce the standard deviation of total returns. Thus, if a delegator votes for a large number of validator candidates, there is a high probability of convergence to the average reward rate. Furthermore, since the expected reward can be calculated by the amount of delegated tokens cast for a validator, it is expected that the average reward will be improved by concentrating votes on a small number of validators with high expected rewards. Consequently, the reward rate is expected to decrease as the number of voting validators increases.

5. Correlation Analysis

5.1. Linear Correlation

In the first analysis, Spearman's rank correlation coefficients are calculated between the reward rates and the four



(a) The number of delegate-related transactions.

(b) The amount of delegated tokens.

(c) The rate of change in the amount of delegated tokens.

(d) The average number of voting validators.

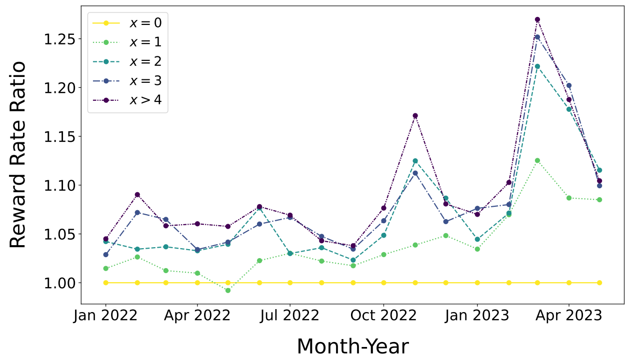
Figure 1. The Spearman's rank correlation coefficients between reward rates and the four attributes of the delegators from January 2022 to May 2023. Correlation coefficients with p-values less than 0.05 are indicated by cross dots. The number of delegate-related transactions and the rate of change in the amount of delegated tokens were correlated with reward rates in most months.

attributes for each month, covering the period from January 2022 to May 2023. Since the four attributes are highly variable across the delegators and do not follow a normal distribution, we decided to use Spearman's rank correlation coefficients. In addition, p-values were also calculated under the null hypothesis that there is no correlation between the two variables. A p-value less than 0.05 indicates the presence of a statistically significant correlation. When calculating the correlation coefficient for the rate of change in the amount of delegated tokens, delegators with a zero rate of change in the amount of delegated tokens were excluded. For reference, a scatter plot of the May 2023 reward rate and the four indicators is included in the Appendix A.

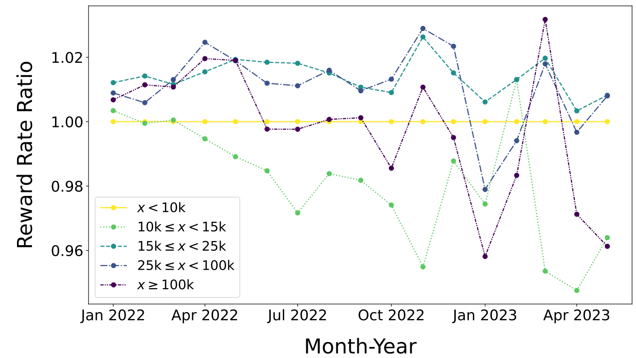
The results are shown in Figure 1. Correlation coefficients with p-values less than 0.05 are indicated by cross dots in the graph. The correlation coefficients for the reward rate and the number of delegate-related transactions (Figure 1a) range from +0.15 to +0.34, indicating a significant correlation across all months. Given the consistently certain absolute values of the correlation coefficients, there is a weak positive correlation between the reward rate and the number of delegate-related transactions. For the reward rate and the amount of delegated tokens (Figure 1b), the correlation coefficients range from −0.07 to +0.07, with only two months showing a significant correlation. The small absolute correlation coefficients and the lack of significance in most months suggest a very weak to no correlation. The correlation coefficients for the reward rate and the rate of change in the amount of delegated tokens (Figure 1c) range from −0.19 to +0.09, with only two months showing a significant correlation. Similar to the result for the amount of delegated tokens, the small absolute coefficients and lack of broad significance suggest a very weak to no correlation. As for the reward rate and the average number of voting validators (Figure 1d), the values range from −0.09 to +0.12, with a significant correlation in half of the months. However, the small absolute coefficients and the lack of consistent significance suggest no linear correlation, or at best a very weak one.

5.2. Non-linear Correlation

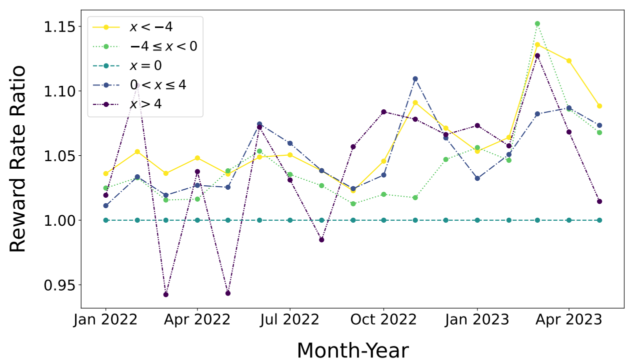
To observe relationships that could not be distinguished by the correlation coefficient alone, delegators were classified into five groups based on attribute values, and the average reward rate ratio was calculated for each group. The delegator classification criteria were as follows: For the number of delegate-related transactions, the groups were



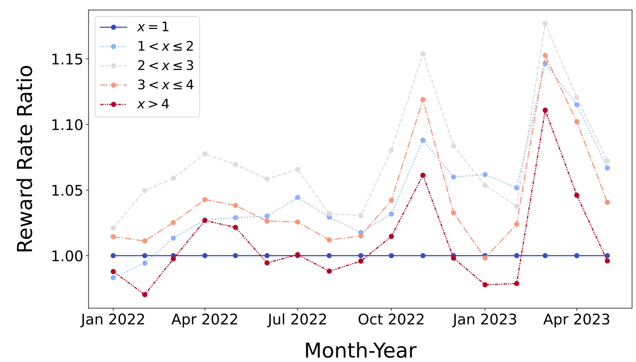
(a) The number of delegate-related transactions.



(b) The amount of delegated tokens.



(c) The rate of change in the amount of delegated tokens.



(d) The average number of voting validators.

Figure 2. The average reward rates grouped by the four attribute values. The reward rates are normalized. A non-linear correlation was observed, with the average reward rate showing a peak in the group with the average number of voting validators between 2 and 3.

0, 1, 2, 3, and 4 or more transactions. For the amount of delegated tokens, the groups were less 10,000, 10,000 to less than 15,000, 15,000 to less than 25,000, 25,000 to less than 100,000, and 100,000 or more. Because the rate of change in the amount of delegated tokens included negative values, we calculated the logarithm of the absolute values to create an index for the grouping. For the rate of change in the amount of delegated tokens, the groups were an index value of less than −4, −4 to less than 0, 0, greater than 0 to 4, and greater than 4. For the average number of voting validators, the groups were 1, greater than 1 to 2, greater than 2 to 3, greater than 3 to 4, and 4 or more. Next, one group was selected as the baseline, and its average reward rate ratio was set to 1. The average reward rate ratios of the other groups were calculated relative to the baseline. These calculations were performed for each month from January 2022 to May 2023.

Figure 2 shows the average reward rates for each group on a monthly basis. Consistent with the analysis in Section 5.1, there is a positive correlation between the reward rate and the number of delegate-related transactions. No correlations were observed between the reward rate and the amount of delegated tokens or the rate of change in the amount of delegated tokens. For the average number of voting validators, the highest reward rate was observed in the $2 < x \leq 3$ group, with a gradual decrease observed around this group, indicating a non-linear correlation. Because it was a non-linear correlation, it was not captured by the Spearman's correlation coefficient.

6. Discussion

When examining the relationship between the reward rate and the number of delegate-related transactions, Figure 1a shows a weak correlation. This finding is consistent with the expectation that the existence of the leveling mechanism requires the generation of delegate-related transactions to maintain a certain minimum reward rate. The results obtained are also consistent with those reported by Tanaka et al. [12].

We could not observe a correlation between the reward rate and the amount of delegated tokens from Figures 1b and 2b. Contrary to expectations, this suggests that the phenomenon of the rich getting richer, often discussed in PoS blockchains, has not occurred in the BNB Chain. However, delegators were identified based on their delegator addresses, which do not necessarily correspond to actual individuals or organizations. If an individual has more than one address, they are recognized as multiple delegators in our analysis. In addition, while the amount of delegated tokens did not correlate with the reward rate for the top 1,000 delegators, the results may be different for the top 10,000 delegators.

A correlation between the reward rate and the rate of change in the amount of delegated tokens was not observed in Figures 1c and 2c. Increasing the amount of delegated tokens would increase the reward, but in calculating the reward rate, the denominator also increases, so simply increasing the amount of delegated tokens does not improve the reward rate. In other words, if delegators increase the amount of delegated tokens, they must vote for validators who can earn more rewards with the increased tokens. Otherwise, the reward rate will not improve. Since it was found from Figure 1b that increasing the amount of delegated tokens does not increase the reward rate, it is assumed that the rate of change in that amount did not increase the reward rate either.

We could not observe a correlation between the reward rate and the average number of voting validators in Figure 1d. However, in Figure 2d, we observed a non-linear correlation, with the highest reward rates for the group with $2 < x \leq 3$, and a gradual decrease around that group. Since delegators can calculate expected rewards by checking the amount of delegated tokens on a validator when voting, concentrating votes on a small number of validators with high expected rewards improves the average reward rate. However, changing where votes are cast is not always free due to cost and frequency constraints. When voting for a large number of validators, a delegator's average reward rate converges to the global average reward rate without changing the voting target, which is why the reward rate gradually decreases when the average number of voting validators is greater than 3. The reward rate also decreases when the average number of voting validators is less than 2, possibly because when the number of voting validators is too small, the reward rate is more likely to decrease without changing the voting target.

7. Conclusion

The BNB Chain using DPoS has a leveling mechanism to prevent the concentration of votes on a small number of validators, but the voting behavior of token holders is not economically rational, resulting in different reward rates. We investigated the factors that affect the delegator's reward rate in the BNB Chain.

We calculated the Spearman's correlation coefficient every month from January 2022 to May 2023 to investigate the relationship between the reward rate and the four delegator attributes: the number of delegate-related transactions, the amount of delegated tokens, the rate of change in the amount of delegated tokens, and the average number of voting validators. We also examined by plotting the average reward rates of delegators grouped by their attributes. As a result, we found that there is a weak positive linear correlation between the reward rate and the number of delegate-related transactions, and that there is a non-linear correlation between the reward rate and the average number of voting validators.

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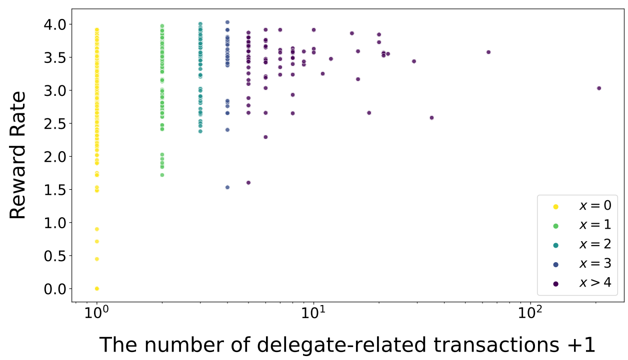
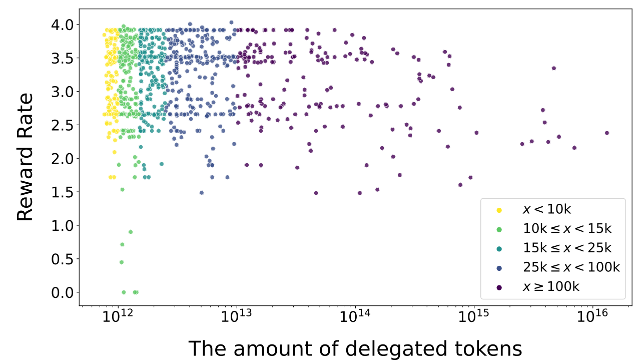
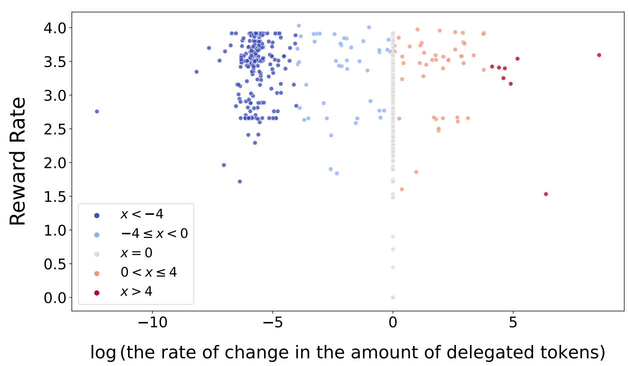


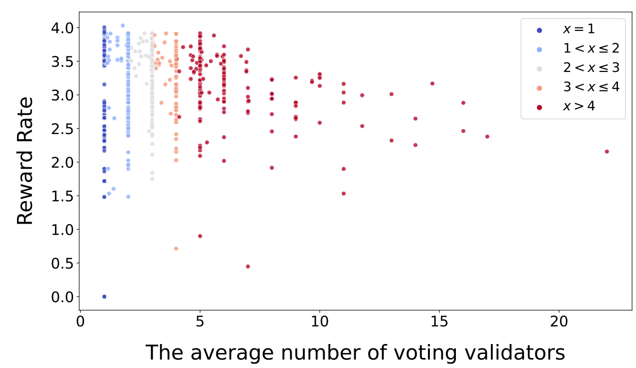
(a) The number of delegate-related transactions.



(b) The amount of delegated tokens.



(c) The rate of change in the amount of delegated tokens.



(d) The average number of voting validators.

Figure 3. Scatter plot between reward rates and the four attributes on May 2023.

Appendix A. Scatter Plot between Reward Rates and Four Attributes

Scatter plots were generated for each month from January 2022 to May 2023, showing the relationship between reward rates and the four attributes of delegators. The delegators were classified into five distinct groups based on their attribute values and plotted using different colors. The number of delegate-related transactions was grouped as follows: 0, 1, 2, 3, and 4 or more. With the amount of delegated tokens, the groups were as follows: 10,000, 10,000 to 15,000, 15,000 to 25,000, 25,000 to 100,000, and 100,000 or more. Because the rate of change in the amount of delegated tokens included negative values, we calculated the logarithm of the absolute values to create an index for the scatter plot. The delegators were subsequently classified into five groups based on this index: those with an index value of less than −4, greater than or equal to −4 and less than 0, 0, greater than 0 and less than or equal to 4, and greater than 4. The average number of voting validators was used to categorize the groups as follows: 1, greater than 1 and less than or equal to 2, greater than 2 and less than or equal to 3, greater than 3 and less than or equal to 4, greater than or equal to 4, and 4 or more. The x axis is plotted on a logarithmic scale for the scatter plot of the number of delegate-related transactions and the amount of delegated tokens. Log-transformed values are also plotted for the rate of change in the amount of delegated tokens.

Due to space limitasions, a scatter plot for May 2023, the latest month in the data, is shown in Figure 3. The number of delegators for each group in Figure 3a was 609 for $x = 0$, 195 for $x = 1$, 67 for $x = 2$, 32 for $x = 3$, and 78 for $x > 4$. The number of delegators for each group in Figure 3b was 140 for $x < 10,000$, 212 for $10,000 \leq x < 15,000$, 197 for $15,000 \leq x < 25,000$, 100 for $25,000 \leq x < 100,000$, and 237 for $x \geq 100,000$. The number of delegators for each group in Figure 3c was 198 for $x < -4$, 44 for $-4 < x < 0$, 676 for $x = 0$, 45 for $x \leq 4$, and 8 for $x > 4$. The number of delegators for each group in Figure 3d was 403 for $x = 1$, 184 for $1 < x \leq 2$, 141 for $2 < x \leq 3$, 79 for $3 < x \leq 4$, and 164 for $x > 4$.

In Figure 3b, all the delegators with a reward rate below 1.0% were included in the $10,000 \leq x < 15,000$ group. In Figure 3c, all the delegators with a reward rate below 1.0% were included in the $x = 0$ group. In Figure 3d, the relationship between the reward rate and the average number of voting validators for the delegators in the $x > 4$ group is observed to converge around 2.5% as the average number of voting validators increases.