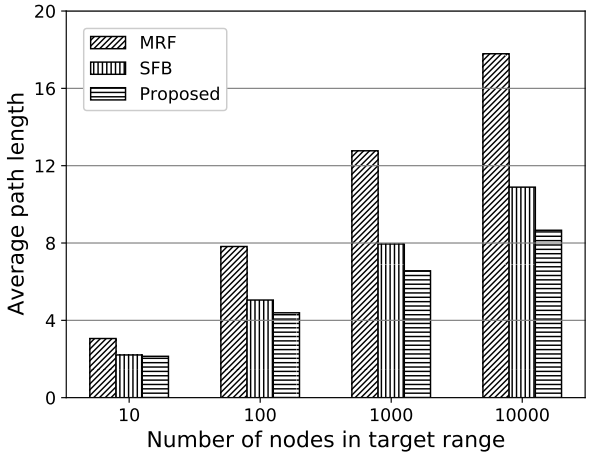


**Fig. 4:** Comparison of the average path length for a topology where keys were generated by a uniform distribution.

### A. Uniform distribution

We generated the keys of the nodes to follow a uniform distribution $P\{v.\text{key} = k\} = \frac{1}{2^{30}} (k \in \{0, 1, ..., 2^{30} - 1\})$ where $v$.key is the key of node $v$. The center estimation $mid$ for this distribution is $mid(k_1, k_2) := \frac{k_1+k_2}{2}$.

Fig. 4 shows the average path length measured by the procedure described above. The proposed method has a shorter average path length than that of the other two methods, although the three routing methods show a similar tendency with respect to the change in $N_R$. In the graph in Fig. 4, compared to SFB, the reduction of the average path length at each $N_R$ is 3.15% ($N_R = 10$), 13.05% ($N_R = 100$), 17.46% ($N_R = 1,000$), and 20.48% ($N_R = 10,000$).

### B. Power-law distribution

We generated the keys of the nodes to follow a power-law distribution $P\{v.\text{key} \leq k\} = \int_0^k f(\kappa)d\kappa \, (0 \leq k \leq 2^{30})$ for a probability density function $f(k) = ck^{10} \, (0 \leq k \leq 2^{30})$. Here, c is a constant that satisfies $\int_0^{2^{30}} f(\kappa)d\kappa = 1$. As mentioned in Section II-A, the average of two keys works well as the center of two nodes in many cases, so the $mid$ used in Section IV-A is also used to estimate the center in this key distribution. The purpose of using a power-law distribution is to evaluate the effect of the proposed method on a biased key distribution. For this purpose, we use the topology from Section IV-A with only the key value changed to observe the change in the average path length for two different key distributions.

Table I compares the results measured by the procedure described above with the results in Section IV-A. For two different key distributions, a comparable path length is observed for each of the three routing methods. For the proposed method, the reduction ratio is equivalent to that in Section IV-A for the two routing methods, indicating that the proposed method is effective, even when the key distribution is biased.

**TABLE I:** Comparison of the average path length for a topology where keys were generated by two different distributions.

| | | $N_R = 10$ | 100 | $1,000$ | $10,000$ |
|---|---|---|---|---|---|
| MRF | key: uniform | 3.06 | 7.82 | 12.77 | 17.79 |
| | key: power | 3.06 | 7.82 | 12.77 | 17.79 |
| SFB | key: uniform | 2.22 | 5.06 | 7.95 | 10.90 |
| | key: power | 2.22 | 5.06 | 7.95 | 10.90 |
| Proposed | key: uniform | 2.14 | 4.40 | 6.56 | 8.67 |
| | key: power | 2.14 | 4.41 | 6.60 | 8.75 |

## V. Conclusion

In this paper, we proposed a new routing method for range queries in Skip Graph, named Detouring Range Search (DRS). By efficiently using the topology of Skip Graph, the proposed method can deliver a query to all nodes in the target range with a minimum number of messages and a short path length. Evaluation experiments show that the proposed method can reduce the average path length by approximately 20% compared to that of existing methods.

We confirmed that the proposed method is effective even when the key distribution is biased. Our future work will include an analytical evaluation of the path length reduction and an investigation of the center estimation methods.

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